

Modeling and Compensating Phase Error in High-Speed 3D Reconstruction

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Abstract. Error-diffusion-based binary-defocusing fringe projection profilometry has been widely adopted for high-speed 3D imaging of dynamic objects due to its high measurement efficiency. However, existing error diffusion methods often neglect the intrinsic relationship between fringe pattern structures and the diffusion direction, resulting in systematic phase errors in the generated binary patterns. To overcome this limitation, we develop a theoretical model of phase errors in the dithering algorithm that elucidates the mechanisms linking phase errors to the diffusion direction. Based on this analysis, a binary fringe generation strategy is designed. In this approach, fringe patterns in the four-step phase-shifting method are grouped, and binarization is performed using both the Floyd Steinberg algorithm and its inverse. This process exploits inter-fringe error-compensation effects to enhance measurement accuracy. Experimental results demonstrate that the proposed method reduces binarization-induced errors by 23.77% without additional computational overhead, confirming its superior performance in terms of both accuracy and efficiency.

Keywords: 3D optical measurement · Phase shifting · Binary defocusing · Phase error correction

1 Introduction

Reconstructing dense 3D geometry from 2D images remains a fundamental problem in computer vision [17, 42, 44], with structured light projection being one of the most accurate and reliable approaches for active 3D sensing. Among various structured light techniques, fringe projection profilometry (FPP) has been widely adopted due to its high spatial resolution and sub-pixel accuracy [7, 25, 28]. However, a key assumption of FPP is that the object remains stationary during one projection cycle. This assumption breaks down in dynamic scenes, where object motion leads to frame misalignment among multiple phase-shifted patterns,

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breaking the ideal phase relationship and introducing motion-induced phase errors. As a result, traditional FPP often produces severe reconstruction distortions in the presence of motion.

To mitigate motion artifacts, several strategies have been explored, including motion estimation and compensation [22], Hilbert-based phase correction [5], and hybrid approaches combining phase-shifting profilometry (PSP) with Fourier transform profilometry (FTP) [8]. Although these methods achieve partial success, they usually assume uniform 2D motion and struggle under complex, nonlinear, or depth-varying motion. When the projection speed significantly exceeds the speed of object, the object can be approximately regarded as static [23]. Hence, increasing the projection rate remains the most direct and robust solution for dynamic 3D measurement. Unfortunately, current digital light processing (DLP) hardware faces substantial limitations in achieving high-frame-rate sinusoidal pattern projection through focused optical systems [4, 47].

To accelerate the measurement speed of FPP, researchers have continuously refined both system hardware and projection strategies. Among these advances, binary defocusing techniques have emerged as a powerful approach to significantly increase the projection rate of optical measurement systems [3]. By projecting defocused binary fringes instead of 8-bit grayscale patterns, this method reduces the information content per projection while avoiding nonlinear gamma distortion, thereby substantially enhancing projector switching speed. Studies have demonstrated that this approach can raise conventional projection rates of around a hundred frames per second to several thousand or even tens of thousands of frames per second [2]. Beyond optimizing fringe patterns, hardware innovations have further contributed to speed improvements. The development of high-speed switching light sources [29], particularly the continuous advancement of LED array technology, has steadily increased both switching frequency and spatial resolution of the system, providing essential support for next-generation high-speed 3D measurement systems. Moreover, the binary on/off nature of LEDs naturally complements binary fringe projection, reinforcing the practical advantages of defocusing techniques. Despite these advances, binary defocusing still faces critical challenges in accuracy. Since binary fringes inherently approximate sinusoidal patterns, high-order harmonic errors are unavoidable and directly affect phase recovery precision. Strong optical defocus can suppress these errors, yet it also reduces fringe contrast and limits measurement accuracy [46]. Among existing binary defocusing approaches, dithering-based algorithms currently achieve the best performance, providing high-fidelity sinusoidal approximation even under shallow defocus. However, the binarization process inevitably introduces residual phase errors, necessitating correction and motivating continued research in system design and fringe optimization strategies.

Building on our prior research on high-speed defocused measurement systems, we propose a novel binarization strategy tailored for four-step phase-shifting method. By leveraging mutual compensation of inter-fringe errors, the method directly achieves high-accuracy phase while maintaining the real-time capability

necessary for high-speed 3D measurement. Its accuracy and effectiveness are confirmed through extensive comparison experiments.

Our main contributions are summarized as follows:

- We theoretically derive the fundamental form of the phase error introduced by error diffusion in dithering-based algorithms and reveal its distinctive properties. Notably, the induced phase error is independent of the number of phase-shifting steps, the fringe period, and the defocusing amount of projector.
- We systematically analyzed the role of the error diffusion kernel during fringe binarization. Experimental results indicate a strong correlation between phase error and the error diffusion direction.
- Based on the above analysis, we propose a robust binarization strategy for four-step phase-shifting method. By applying error diffusion kernels with different orientations to the fringe patterns, the approach achieves inter-fringe self-compensation. Compared to existing optimization methods, the phase error is reduced by approximately 23.77%.

2 Related Work

Fringe Projection Strategies. One of the primary directions to enhance measurement efficiency in FPP is the optimization of projection strategies [2, 3, 9, 10, 50]. Zhu *et al.* proposed a color-encoded fringe technique, where multiple fringe patterns are composited into a single color pattern to reduce the number of required projections and accelerate measurement [50]. Similarly, Chen *et al.* introduced a one multiplexed fringe pattern captured with a single long exposure [2]. By embedding Fourier transform, phase shifting, and ensemble learning into a deep neural network, they decomposed the pattern and extracted individual fringe components. Further extending this idea, Chen *et al.* encoded multi-time frame 3D information into a single multiplexed pattern and reconstructed high-precision absolute phase using two-stage number-theoretical-based deep neural networks [3]. Han *et al.* developed a phase-shifting superimposition projection framework to reconcile the frame rate mismatch between the camera acquisition and projector projection processes [10].

High-Speed Systems and Motion Calibration. To address dynamic measurement challenges, researchers have developed high-speed optical setups and motion compensation models [11, 13–15, 32]. Johnson *et al.* designed a high-speed projection system via a pair of single-mode optical fibers mounted within a needle, achieving sub-millimeter depth profiling at 60 frames per second [15]. Similarly, Huang *et al.* implemented a self-developed 128×128 high-speed LED array with a maximum refresh rate of 25 MHz for binary fringe patterns, resulting in a 1000-fold speed enhancement over DMD [13]. In terms of motion error correction, Jeon *et al.* address the mismatch problem due to motion-induced camera pixel disparities by assuming constant motion and using the pinhole camera model [14]. Wang *et al.* incorporated speckle patterns into projection sequences to assist motion estimation [32], while He *et al.* aligned the isophase

direction of fringe patterns with the known direction of the moving objects to minimize phase distortion [11].

Binary Defocusing Algorithm. The foundation of binary defocusing can be traced back to [18], which observed that light passing through venetian blinds naturally forms sinusoidal distributions, implying that optical defocus can transform binary patterns into smooth sinusoidal fringes. This insight inspired a series of binary defocusing algorithms widely applied in high-speed 3D measurement [12, 16, 19, 39]. To further enhance performance, pulse-width modulation (PWM) is introduced to suppress high-order harmonics [1], followed by optimized PWM (OPWM) [34] and triple-SPWM [51]. Currently, binary encoding approaches can be broadly categorized into dithering-based and iterative optimization-based strategies. The former includes ordered [6], random [35], and error diffusion dithering [33], with the Floyd Steinberg (FS) [30] and Sierra Lite [31] algorithms serving as classical examples. Several studies have improved upon these by expanding or tuning the error diffusion kernel to reduce harmonic distortion [26, 37, 48]. Based on phase-shifting error models, You *et al.* further discovered phase-error invariance under blur and applied fixed phase compensation to enhance reconstruction precision [41]. Additionally, You *et al.* proposed the half-truncated sinusoidal fringes, in which half of the intensity values are set to zero, to simplify the binarization and improve phase stability [40]. Furthermore, iterative optimization of defocused fringe gray levels has also been explored to further improve measurement precision [21, 36].

3 Preliminary Knowledge

A conventional FPP system typically consists of a digital projector and a camera arranged in a fixed stereo-configuration, as illustrated in Fig. 1. By treating the projector as an inverse camera, the system forms a measurement model analogous to that of stereo vision [24]. Its fundamental principle involves projecting predefined fringe patterns onto the object surface, where each surface point is uniquely encoded and captured by the camera. This process significantly simplifies pixel correspondence between the projector and the camera. Among various encoding strategies, PSP is the most widely used, which requires projecting a series of sinusoidal fringe patterns.

$$I_n(u, v) = A + B \cos[\phi(u, v) + \delta_n], \quad n = 1, 2, \dots, N, \quad (1)$$

where, I_n represents the n_{th} phase-shifting fringe pattern; (u, v) denotes the pixel coordinate on the imaging plane of the projector; A and B denote the average intensity and intensity modulation, respectively; $\delta_n = 2\pi(n - 1)/N$ defines the amount of phase shift; and N is the total number of phase-shifting patterns. The wrapped phase ϕ can be unwrapped to obtain the absolute phase Φ , which represents the unique encoded information in PSP. Once Φ is recovered, it can be combined with the calibrated parameters of the system to achieve precise 3D

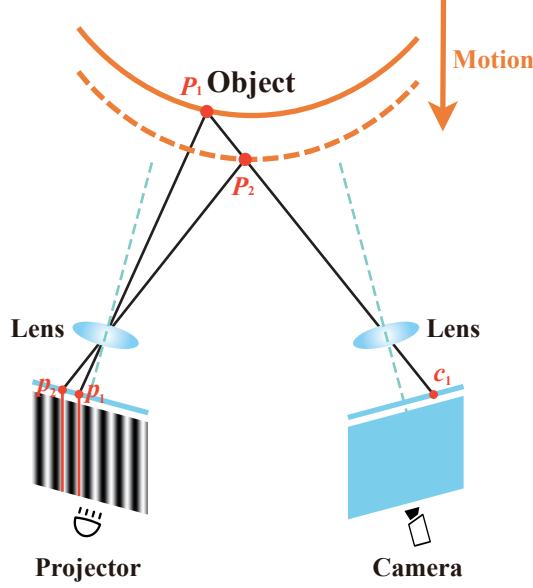


Fig. 1: Imaging schematic for moving objects.

reconstruction. The wrapped phase ϕ is calculated as follows:

$$\phi(u, v) = \arctan \frac{-\sum_{n=1}^N \sin(\delta_n) I_n(u, v)}{\sum_{n=1}^N \cos(\delta_n) I_n(u, v)} = \arctan \frac{\sin(\phi(u, v))}{\cos(\phi(u, v))}. \quad (2)$$

This formulation highlights that the precision of the projected fringe patterns is crucial for the accurate recovery of phase. When measuring dynamic objects, additional distortions arise due to motion. As illustrated in Fig. 1, during a single exposure, the intensity of a camera pixel, such as c_1 , may shift from the corresponding projector pixel p_1 to p_2 , resulting in blurred fringe patterns. Furthermore, the temporal intervals between successive projected patterns convert object motion into more complex phase inconsistencies across the sequence. To mitigate these motion-induced errors, high-frame-rate binary-defocusing FPP is commonly employed.

The binary-defocusing FPP algorithm quantizes each pixel intensity to either 0 or 255 according to a predefined binarization strategy, thereby approximating ideal sinusoidal fringes. Fig. 2 illustrates the fundamental procedure of the FS dithering algorithm. For clarity, pattern blocks are used in this figure to represent individual pixels, where blue dots indicate the pixels currently undergoing binarization, and yellow dots denote adjacent unprocessed pixels. FS algorithm traverses the pattern from left to right and top to bottom, sequentially binarizing each pixel. The core principle is that, once a pixel is binarized, the resulting intensity error is propagated to neighboring unprocessed pixels according to a predefined set of parameters, thereby preserving the local intensity consistency

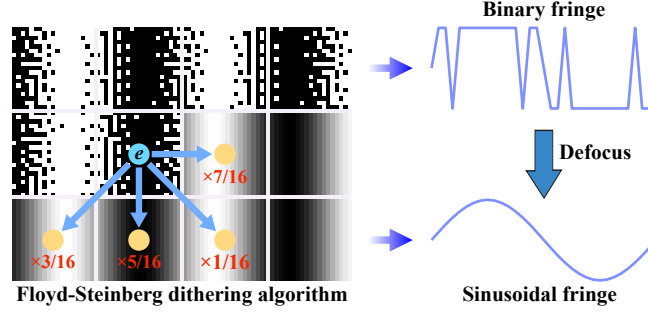


Fig. 2: FS dithering algorithm.

of the simulated fringe pattern. This error propagation can be mathematically represented by an error diffusion kernel as follows:

$$k = \frac{1}{16} \begin{pmatrix} - & \bullet & 7 \\ 3 & 5 & 1 \end{pmatrix}, \quad (3)$$

where, the symbol $-$ denotes previously processed pixels, while $\bullet$ marks the pixel currently undergoing processing. By employing this diffusion kernel, the FS algorithm generates a binarized approximation of the sinusoidal fringe patterns, with the resulting cross-sectional intensity profile presented on the right side of Fig. 2. Subsequently, following the fundamental principle of binary-defocusing FPP, the projected binary fringes can be optically smoothed into approximate sinusoidal fringes by projector defocusing. The complete mathematical model describing this process is presented as follows:

$$\begin{cases} I'_n(u, v) = I_n(u, v) + \sum_{\Omega} k(i, j)e(x - i, y - j), \\ I_n^b(u, v) = \begin{cases} 0, & I'_n(u, v) < 127.5 \\ 255, & I'_n(u, v) \geq 127.5 \end{cases}, \\ e(u, v) = I'_n(u, v) - I_n^b(u, v), \\ I_n(u, v) = h(u, v) * I_n^b(u, v). \end{cases} \quad (4)$$

where, $I_n(u, v)$ represents the original grayscale intensity before binarization, while $I'_n(u, v)$ denotes the modified intensity after incorporating the propagated quantization errors from neighboring pixels. $k(i, j)$ defines the weighting coefficients of the error diffusion kernel, and Ω refers to the set of previously processed pixels that contribute error to the current pixel (u, v) . The binarized fringe of FS algorithm is denoted as $I_n^b(u, v)$, and the residual quantization error at (u, v) is expressed as $e(u, v)$. During the optical projection process, the defocusing effect of the projector can be approximated by a point spread function (PSF), denoted as h , which is typically modeled using a two-dimensional Gaussian function [26, 41]. This PSF acts as an optical low-pass filter on the projected fringe patterns, causing the reconstructed fringes to exhibit a lower signal-to-noise ratio compared to the ideal sinusoidal patterns.

4 Error Analysis and Correction Method

This section begins with a theoretical derivation of the explicit form of the phase error. Based on the resulting error model, we further analyze its distinctive characteristics and subsequently develop a tailored correction algorithm.

4.1 Error Analysis

Although numerous studies have confirmed the presence of residual phase errors in the restored fringes, analytically characterizing these errors remains challenging due to the inherent complexity of Eq. (4). To mitigate this difficulty, existing optimization strategies have primarily concentrated on refining the structure or tuning the parameters of the error diffusion kernel. However, their key limitation lies in neglecting the influence of the diffusion direction. As illustrated in Fig. 2, because the sinusoidal fringe maintains a uniform gray-level distribution along the vertical direction, the error diffusion process predominantly manifests as horizontal intensity propagation. Based on this observation, it is reasonable to assume that error diffusion mainly affects the lateral intensity distribution of the fringes, whereby the restored fringe pattern can be reformulated as:

$$I_n = a + b \cos(\phi + \delta_n) + k_1 \cos(\phi + \delta_n - \frac{2\pi}{P}) + k_2 \cos(\phi + \delta_n + \frac{2\pi}{P}). \quad (5)$$

For simplicity, the pixel coordinates (u, v) are omitted. a and b denote the average intensity and the modulation amplitude after defocusing, respectively, while k_1 and k_2 are variables determined by the error diffusion parameters. Further derivation leads to:

$$I_n = a + \alpha \cos(\phi + \delta_n) \cos \frac{2\pi}{P} + \beta \sin(\phi + \delta_n) \sin \frac{2\pi}{P}, \quad (6)$$

where, α and β represent combined trigonometric coefficients, and P denotes the period of the fringe pattern. The sine component corresponds to the intensity error ΔI_n present in the restored fringe. The relationship between the phase error and the fringe intensity error ΔI_n can be expressed as follows:

$$\begin{cases} S = -\sum_{n=1}^N \sin(\delta_n) I_n \\ C = \sum_{n=1}^N \cos(\delta_n) I_n \end{cases}. \quad (7)$$

$$\left. \frac{\partial \phi}{\partial I_n} \right|_{\Delta I_n=0} = \frac{1}{C^2 + S^2} \times \left(C \frac{\partial S}{\partial I_n} - S \frac{\partial C}{\partial I_n} \right) \Big|_{\Delta I_n}. \quad (8)$$

According to the existing intensity noise error model [20], Eq. (8) is transformed into:

$$\left. \frac{\partial \phi}{\partial I_n} \right|_{\Delta I_n=0} = \frac{2}{N\alpha} \sin(\phi + \delta_n) / \cos \frac{2\pi}{P}. \quad (9)$$

By regarding the sinusoidal component as the intensity error ΔI_n and substituting it into Eq. (9), the following expression is obtained:

$$\begin{aligned}\Delta\phi &= \frac{2}{N\alpha} \sum_{n=1}^N \sin(\phi + \delta_n) / \cos \frac{2\pi}{P} \times \Delta I_n \\ &= \frac{2}{N\alpha} \sum_{n=1}^N \sin(\phi + \delta_n) \times \beta \sin(\phi + \delta_n) \tan \frac{2\pi}{P} = \frac{\beta}{\alpha} \tan \frac{2\pi}{P}.\end{aligned}\quad (10)$$

This is the error model proposed in this paper. According to this model, the phase error introduced by the error diffusion algorithm is independent of the number of phase-shifting steps N . Additionally, this error remains unaffected by variations in the defocusing. Leveraging the properties of the Gaussian function, the impact of additional defocusing can be described by another Gaussian function, h_1 . The expression for the fringe pattern after additional defocusing is [38]:

$$\begin{aligned}I_n^h &= h_1 * I_n = H_1(0, 0)a + H_1(\omega_0, 0) \times \\ &\quad [\alpha \cos(\phi + \delta_n) \cos \frac{2\pi}{P} + \beta \sin(\phi + \delta_n) \sin \frac{2\pi}{P}],\end{aligned}\quad (11)$$

where, H_1 denotes the Fourier transform of h_1 , and $\omega_0 = 2\pi/P$ represents the angular frequency of the fringe pattern. Eq. (10) is rederived based on this formulation, yielding the same result. Consequently, when defocus is sufficient, the error is independent of the projector defocus. In this section, horizontal fringes are utilized, allowing Eq. (10) can be transformed into a horizontal coordinate error of the projector. Notably, when the fringes are wide ($P > 24$ pixels, as experimentally verified), $2\pi/P \rightarrow 0$, and the horizontal coordinate error becomes constant:

$$\lim_{P \rightarrow \infty} \Delta u = \Delta\phi \times \frac{P}{2\pi} = \frac{\beta}{\alpha}.\quad (12)$$

The theoretical derivation demonstrates that the phase error is independent of these three parameters. The underlying reason is that the intensity error in Eq. (6) shares the same frequency as the ground truth, which differs from conventional intensity noise. If this error can be eliminated by adjusting these parameters, the ground truth component will also be eliminated. This derivation has been observed and validated in many studies [27, 41, 49], confirming the accuracy and reliability of the proposed model. Detailed analysis involving multiple periods and defocuses can be found in the **Supplementary Materials**.

4.2 Algorithm Design

In Eq. (5), k_1 and k_2 represent the grayscale error coefficients associated with forward and backward propagation, respectively. Their values depend on the specific configuration of the error diffusion kernel. As illustrated in Fig. 3(a), when an error diffusion kernel that is exactly the inverse of the FS kernel is

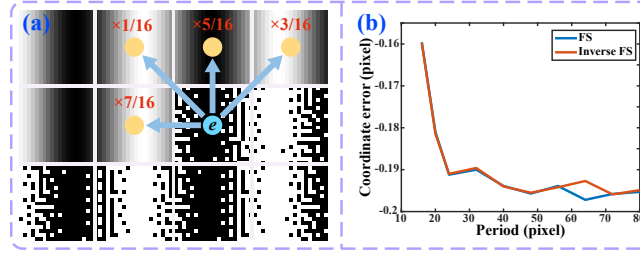


Fig. 3: (a) Schematic of the inverse FS dithering algorithm; (b) Line graphs of showing the error variation of FS and inverse FS algorithm.

employed, the values of k_1 and k_2 are theoretically expected to be exchanged. Under this condition, the expression of I_n should be updated as follows:

$$\begin{aligned} I_n &= a + b \cos(\phi + \delta_n) + k_2 \cos(\phi + \delta_n - \frac{2\pi}{P}) + k_1 \cos(\phi + \delta_n + \frac{2\pi}{P}) \\ &= a + \alpha \cos(\phi + \delta_n) \cos \frac{2\pi}{P} - \beta \sin(\phi + \delta_n) \sin \frac{2\pi}{P}. \end{aligned} \quad (13)$$

Accordingly, the resulting phase error can be expressed as:

$$\Delta\phi = -\frac{\beta}{\alpha} \tan \frac{2\pi}{P}. \quad (14)$$

To validate the above conclusion, we evaluate the average phase errors generated by the FS and inverse FS diffusion kernels under varying fringe periods. For ease of comparison, the phase errors from the inverse FS algorithm are inverted and plotted alongside those from the standard FS algorithm in Fig. 3(b). It can be observed that the amplitudes of the phase errors from both approaches are essentially identical, which aligns well with the theoretical derivation.

To further assess the accuracy of the grayscale variation modeled in Eq. (5), a fringe pattern with a period of 16 pixels is selected as a representative example. This pattern is binarized using both the standard FS and the inverse FS (IFS) method, as illustrated in Figs. 4(a) and (c), respectively. For reference, the original 8-bit sinusoidal fringe pattern (serving as the ground truth) is displayed in Fig. 4(b). To simulate the defocusing effect of the projector, a Gaussian filter with a standard deviation of 3 is applied uniformly to the three fringe patterns to ensure overall consistency in their intensity variations. The resulting patterns are shown in Figs. 4(d-f). The intensity errors, illustrated in Figs. 4(g) and (h), are calculated by subtracting Figs. 4(d) and (f) from Fig. 4(e), respectively. Fig. 4(i) presents a detailed comparison of the single-line errors between the FS and IFS algorithms. As shown in Fig. 4(i), the intensity error distribution produced by the inverse FS is nearly the mirror opposite of that generated by the standard FS. This observation corroborates the theoretical analysis and confirms that Eq. (10) is an accurate error model.

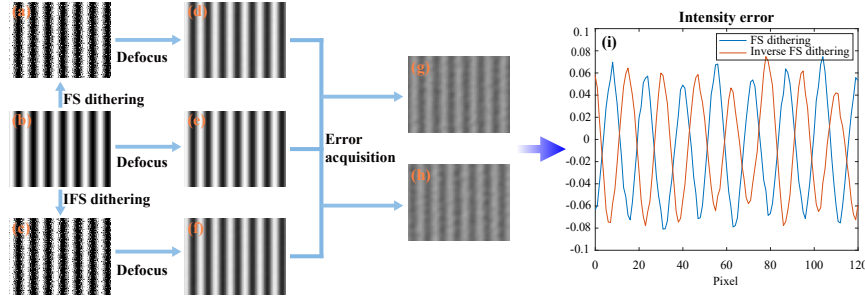


Fig. 4: Comparison of intensity errors between FS and inverse FS for a 16-period fringe pattern. (a) FS binary pattern; (b) Standard 8-bit sinusoidal pattern; (c) Inverse FS binary pattern; (d) Defocused FS pattern; (e) Defocused sinusoidal pattern; (f) Defocused inverse FS pattern; (g) Intensity error of FS binary pattern; (h) Intensity error of inverse FS binary pattern; (i) Detailed errors of FS and inverse FS binary patterns.

As indicated in Eq. (6), the dithering algorithm introduces errors that increase the number of unknown variables in the captured fringe patterns to five: a , $\alpha \cos(\phi)$, $\alpha \sin(\phi)$, $\beta \cos(\phi)$, and $\beta \sin(\phi)$. Accordingly, compensating for the induced errors would theoretically require a five-step phase-shifting method. To improve the efficiency of the algorithm, this section proposes a novel binarization strategy based on a four-step phase-shifting method, as illustrated in Fig. 5. Specifically, the conventional 8-bit four-step phase-shifting sinusoidal fringe patterns are divided into two groups: the first two and the last two patterns. One group is binarized using the standard FS dithering algorithm, while the other group is processed using its inverse algorithm. These binary patterns are subsequently defocused and projected onto the object surface, and the corresponding patterns are captured by a synchronized camera. Under these conditions, these fringe patterns can be mathematically expressed as follows:

$$\begin{cases} I_1 = a + \beta \sin(\phi) \sin \frac{2\pi}{P} + \alpha \cos(\phi) \cos \frac{2\pi}{P} \\ I_2 = a + \beta \cos(\phi) \sin \frac{2\pi}{P} - \alpha \sin(\phi) \cos \frac{2\pi}{P} \\ I_3 = a + \beta \sin(\phi) \sin \frac{2\pi}{P} - \alpha \cos(\phi) \cos \frac{2\pi}{P} \\ I_4 = a + \beta \cos(\phi) \sin \frac{2\pi}{P} + \alpha \sin(\phi) \cos \frac{2\pi}{P} \end{cases} \quad (15)$$

The phase is then retrieved utilizing the conventional four-step phase-shifting algorithm, with the wrapped phase calculated as:

$$\phi = \arctan \frac{I_4 - I_2}{I_1 - I_3}. \quad (16)$$

Notably, the proposed optimization algorithm employs a carefully designed binarization strategy, which reduces the number of unknowns in Eq. (15) to four: $a + \beta \cos(\phi) \sin \frac{2\pi}{P}$, $a + \beta \sin(\phi) \sin \frac{2\pi}{P}$, $\alpha \cos(\phi) \cos \frac{2\pi}{P}$, and $\alpha \sin(\phi) \cos \frac{2\pi}{P}$. Moreover, a simple subtraction operation is used to eliminate the first two interference terms. This process effectively transforms the dithering-induced error

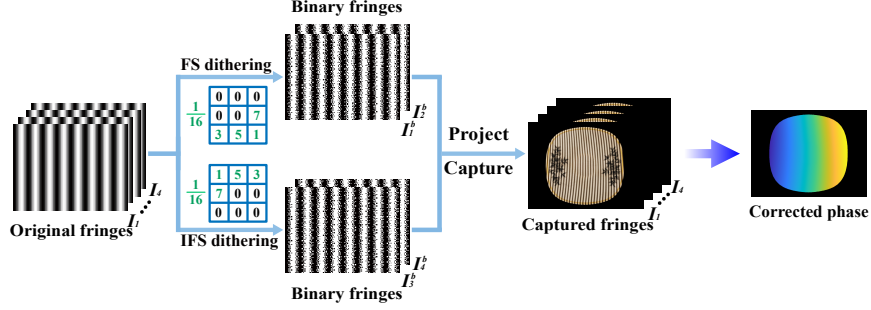


Fig. 5: Flowchart of the Error Diffusion Fusion method.

into a fixed coefficient $\cos \frac{2\pi}{P}$ within the residual term, which can then be removed via division, thereby enabling accurate phase retrieval. Furthermore, by substituting the intensity error from Eq. (15) into Eq. (10) for analytical derivation of the phase error, the result further confirms the above conclusion. Since the proposed method operates solely on the generation of binary fringe patterns, it introduces no additional time overhead during actual measurements, making it a highly efficient preprocessing strategy. Fundamentally, this approach fuses the binarization results obtained from both the FS error diffusion kernel and its inverse. Therefore, in this paper, it is referred to as the Error Diffusion Fusion (EDF) method.

5 Experiment

To further validate the effectiveness of the proposed method in practical measurement, an FPP-based 3D measurement system is established. The system consists of a computer, a projector (DLP LightCrafter 4500; Texas Instruments, Inc., USA), and a camera (acA640-750um; Basler Vision Technology, Inc., Germany). The resolution of the projector is 912×1140 pixels, and its maximum projection rates are 120 Hz at 8-bit mode and 4225 Hz at binary mode. The resolution of the camera is 640×480 pixels, and its maximum frame rate is 751 fps at normal sensor readout mode. The system is 0.7 to 1 m away from the object to be measured, the angle between the optical axis of the projector and that of the camera is approximately 30° . The system is calibrated by Zhang's method [45] based on four-step phase-shifting method with CGC method [43]. In addition to the FS algorithm and EDF method, four existing methods are used to binarize fringes with a period of 16 pixels. These methods include the Optimized Kernel (OK) algorithm [26], Error Fitting (EF) algorithm [12], Half-Truncated Fringe (HTF) algorithm [40], and Fixed Offset Compensation (FOC) algorithm [41]. The ground truth is acquired using 8-bit grayscale sinusoidal fringes with the same period and phase-shifting steps, thereby avoiding binarization-induced bias. To ensure the objectivity of the comparison, all experimental conditions remain consistent. To compare the reconstruction accuracy and robustness, three com-

parison experiments are designed. The first experiment evaluates smooth objects. The second experiment targets non-smooth complex-shaped objects. The third experiment assesses dynamic objects. For more experiments and details, please refer to the **Supplementary Materials**.

5.1 Experiment on Smooth Objects

The first experiment aims to assess the performance of the proposed method on conventional smooth-surfaced objects. In this experiment, the projector is slightly defocused, and binary fringe patterns generated by various methods are projected to measure three representative objects: a stepped block, a horse model, and a puppy sculpture. These objects encompass typical geometric features, such as planar areas and smooth curvatures. During measurement, the projected fringe patterns are modulated by the object surfaces and subsequently captured by the camera. The first column of Fig. 6 presents the appearance of the measured objects, and the second column displays the corresponding reconstructed point clouds obtained by FS. The point cloud generated from 8-bit sinusoidal fringes serves as the ground truth for quantitative comparison. The remaining six columns illustrate the error distribution for all methods.

The FS algorithm introduces significant directional offset errors, whereas the other five optimized methods effectively mitigate these errors. To enable a comprehensive comparison, both average error (AE) and the root mean square error (RMSE) are calculated, as summarized in Table 1. In the table, the lowest

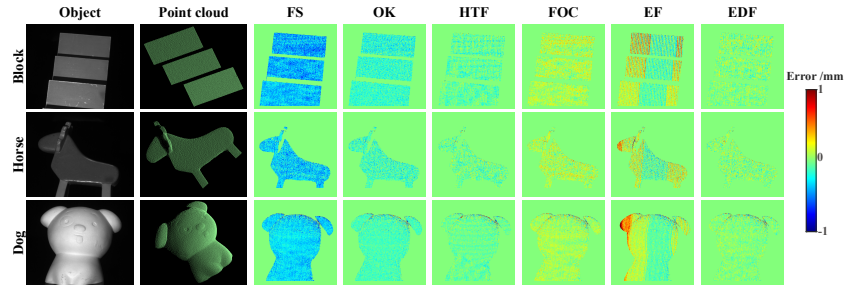


Fig. 6: The measured smooth objects and reconstructed point clouds, together with the corresponding error distributions recovered by different methods. The acronyms in the columns denote Floyd Steinberg (FS), Optimized Kernel (OK), Half-Truncated Fringe (HTF), Fixed Offset Compensation (FOC), Error Fitting (EF), and the proposed Error Diffusion Fusion (EDF) method.

Table 1: AE / RMSE of the measured smooth objects (Unit: mm).

Method	FS	OK	HTF	FOC	EF	EDF
Block	0.218/0.311	0.099/0.158	0.066 /0.116	0.073/ 0.124	0.108/0.186	0.064 / 0.109
Horse	0.105/0.210	0.051/0.115	0.041 /0.191	0.046/ 0.107	0.066/0.180	0.041 / 0.095
Dog	0.164/0.248	0.073/0.130	0.055 /0.212	0.061/ 0.116	0.082/0.161	0.053 / 0.104

and second-lowest errors are highlighted in red and blue, respectively. The HTF method achieves a relatively low AE but exhibits from a higher RMSE, primarily due to the local distortions introduced by half-truncated fringes near object boundaries. The FOC method employs a fixed phase compensation of 0.19 pixels. However, as indicated by Eq. (10), this assumption fails for fringe patterns with short periods, leading to overcompensation. In contrast, the proposed EDF method consistently delivers more robust and stable performance across different conditions. Compared to FS, it achieves a 72.22% reduction in AE and a 58.30% reduction in RMSE.

5.2 Experiment on Non-smooth Objects

The second experiment is designed to evaluate the performance of the proposed method on irregular objects, providing insight into the robustness of various optimization approaches. The experiment employs the same system setup as the first one and measures six objects featuring abrupt depth variations, including a deer sculpture, a metal caster, a chip, and three portrait sculptures.

The results are presented in Fig. 7, where the first column depicts the appearance of the measured objects, the second column displays the point clouds reconstructed by FS method, and the following five columns illustrate the error distributions for the different algorithms. Detailed quantitative results are summarized in Table 2. It is evident that the EDF method exhibits outstand-

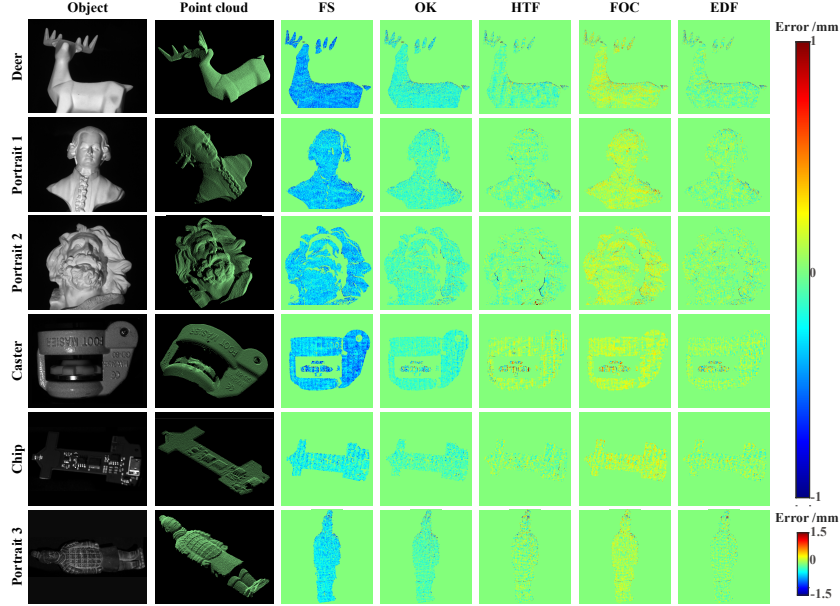


Fig. 7: The measured non-smooth objects and reconstructed point clouds, as well as the corresponding error distributions recovered by various methods.

Table 2: AE / RMSE of the measured non-smooth objects (Unit: mm).

Method	FS	OK	HTF	FOC	EDF
Deer	0.147/0.358	0.068/0.274	0.052 /0.596	0.052/ 0.259	0.047 / 0.114
Portrait 1	0.133/0.236	0.055/0.119	0.043 /0.185	0.050/ 0.110	0.042 / 0.091
Portrait 2	0.173/0.282	0.073/0.170	0.090/0.896	0.071 / 0.167	0.057 / 0.155
Caster	0.193/0.360	0.074/0.237	0.126/1.825	0.062 / 0.228	0.049 / 0.220
Chip	0.104/0.199	0.062/0.171	0.062 /0.680	0.065/ 0.135	0.051 / 0.110
Portrait 3	0.272/0.581	0.142/0.664	0.165/1.621	0.118 / 0.462	0.107 / 0.273

ing robustness, particularly for the deer model and portrait 3, where its RMSE is substantially lower than those of competing methods. Furthermore, even in measurements involving the highly complex geometries of the other objects, EDF consistently achieves the lowest errors. Overall, EDF delivers superior reconstruction accuracy, reducing the AE by 13.85% and the RMSE by 23.77% relative to the second-best method.

5.3 Experiment on Dynamic Objects

In the previous experiments, 8-bit sinusoidal fringes were used as the ground truth to measure static objects and analyze the intrinsic errors of each algorithm. In this section, dynamic objects are measured to evaluate the robustness of the algorithms. Since the optimization algorithms discussed above focus on fringe accuracy, their speed improvements are primarily derived from binary projection, with no fundamental differences between them. Therefore, this section compares the high-speed performance of the EDF and 8-bit fringe projection algorithms. Fig. 8 illustrates the comparison, where a robotic arm moves the object and measurements are taken at five different positions. The result from 8-bit fringes shows distortion due to speed limitations, while EDF performs well, proving EDF enhances system robustness with binary projection.

Furthermore, to evaluate the computational efficiency of the various algorithms, Table 3 summarizes the additional runtime overhead introduced by each method during the reconstruction process. **Acquisition** refers to the pattern acquisition time, which is the primary focus of optimization for the binary-defocusing FPP. Currently, this is limited by the camera used in this paper, but

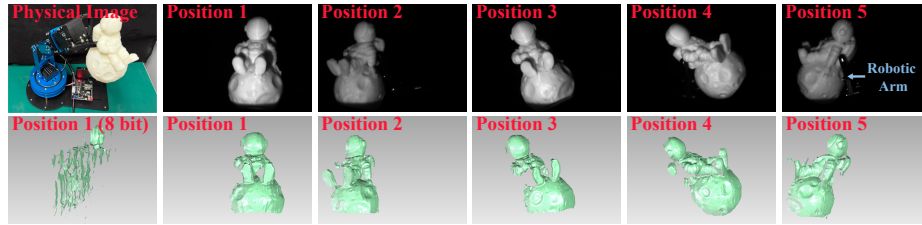
**Fig. 8:** Dynamic scene: 8-bit sinusoidal fringes result and EDF results.

Table 3: The time consumption of each algorithm (Unit: s).

Method	Acquisition	FS	OK	HTF	FOC	EF	EDF
Time	0.009	0	0	0	0.007	0.085	0

it could theoretically reach 0.001 s. FS, OK, HTF, and the proposed EDF are pre-correction algorithms that operate during the fringe pattern generation stage. Thus, they incur no runtime overhead during reconstruction. Conversely, FOC and EF are post-correction algorithms executed after phase retrieval, resulting in additional computation. Overall, EDF achieves the lowest computational overhead while delivering highest accuracy. This balance makes it especially suitable for real-time or resource-constrained applications.

6 Discussion and Limitations

The proposed EDF strategy is mainly implemented within the four-step phase-shifting framework. For the under-constrained three-step case, a possible extension is to adopt a double-binarization strategy, where the fringe patterns are binarized using both the forward and inverse diffusion kernels. This strategy can partially cancel the opposite phase biases, but may also introduce additional high-order harmonics. In future work, we will further elaborate on this extension and develop a more general EDF method for odd-step phase-shifting frameworks.

7 Conclusion

To address the systematic phase errors inherent in existing error diffusion methods, this paper establishes a fundamental theoretical model of phase errors and elucidates the intrinsic relationship between phase errors and the diffusion direction. Based on this theoretical analysis, a robust binary fringe generation strategy tailored for the four-step phase-shifting method is proposed, which achieves high-precision 3D measurements by leveraging inter-fringe error-compensation effects. Experimental results show that the proposed method effectively suppresses measurement errors, reducing the average error (AE) by 13.85% and the root mean square error (RMSE) by 23.77%, outperforming existing mainstream approaches in accuracy while incurring no additional computational overhead. These findings validate both the effectiveness of the theoretical model and the practical value of the proposed optimization strategy.

Acknowledgements

This work was supported by the National Natural Science Foundation of China (62573119), the Major Science and Technology Project of Jiangsu Province (BG2024003), and the Special Project on Basic Research of Frontier Leading Technology of Jiangsu Province of China (BK20192004C).

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